6.2.1 Exercises

For a link to all of the additional resources available for this section, click OSttS Chapter 6 materials.

For help with Exercises 1 - 29 click one or more of the resources below:

- Understanding the Power Rule of Logarithms
- Understanding the Product Rule of Logarithms
- Understanding the Quotient Rule of Logarithms
- Understanding the Change of Base of Logarithms
- Using the Properties of Logarithms

In Exercises 1 - 15, expand the given logarithm and simplify. Assume when necessary that all quantities represent positive real numbers.

1. $\ln(x^3y^2)$
2. $\log_2\left(\dfrac{128}{x^2+4}\right)$
3. $\log_5\left(\dfrac{z}{25}\right)^3$
4. $\log(1.23 \times 10^{37})$
5. $\ln\left(\dfrac{\sqrt{z}}{xy}\right)$
6. $\log_5\left(x^2-25\right)$
7. $\log_{\sqrt{2}}\left(4x^3\right)$
8. $\log_{\frac{1}{3}}(9x(y^3-8))$
9. $\log\left(1000x^3y^5\right)$
10. $\log_3\left(\dfrac{x^2}{81y^4}\right)$
11. $\ln\left(\sqrt[4]{\dfrac{xy}{ez}}\right)$
12. $\log_6\left(\dfrac{216}{x^3y}\right)^4$
13. $\log\left(\dfrac{100x\sqrt{y}}{\sqrt[3]{10}}\right)$
14. $\log_{\frac{1}{2}}\left(\dfrac{4\sqrt[3]{x^2}}{y\sqrt{z}}\right)$
15. $\ln\left(\dfrac{\sqrt[3]{x}}{10\sqrt{yz}}\right)$

In Exercises 16 - 29, use the properties of logarithms to write the expression as a single logarithm.

16. $4\ln(x)+2\ln(y)$
17. $\log_2(x)+\log_2(y)-\log_2(z)$
18. $\log_3(x)-2\log_3(y)$
19. $\frac{1}{2}\log_3(x)-2\log_3(y)-\log_3(z)$
20. $2\ln(x)-3\ln(y)-4\ln(z)$
21. $\log(x)-\frac{1}{3}\log(z)+\frac{1}{2}\log(y)$
22. $-\frac{1}{3}\ln(x)-\frac{1}{3}\ln(y)+\frac{1}{3}\ln(z)$
23. $\log_5(x)-3$
24. $3-\log(x)$
25. $\log_7(x)+\log_7(x-3)-2$

26. $\ln(x) + \frac{1}{2}$

27. $\log_2(x) + \log_4(x)$

28. $\log_2(x) + \log_4(x-1)$

29. $\log_2(x) + \log_{\frac{1}{2}}(x-1)$

In Exercises 30 - 33, use the appropriate change of base formula to convert the given expression to an expression with the indicated base.

30. 7^{x-1} to base e

31. $\log_3(x+2)$ to base 10

32. $\left(\frac{2}{3}\right)^x$ to base e

33. $\log(x^2+1)$ to base e

In Exercises 34 - 39, use the appropriate change of base formula to approximate the logarithm.
For help with these exercises, click the resource below:

- Understanding the Change of Base of Logarithms

34. $\log_3(12)$

35. $\log_5(80)$

36. $\log_6(72)$

37. $\log_4\left(\frac{1}{10}\right)$

38. $\log_{\frac{3}{5}}(1000)$

39. $\log_{\frac{2}{3}}(50)$

40. Compare and contrast the graphs of $y = \ln(x^2)$ and $y = 2\ln(x)$.

41. Prove the Quotient Rule and Power Rule for Logarithms.

42. Give numerical examples to show that, in general,

    (a) $\log_b(x+y) \neq \log_b(x) + \log_b(y)$

    (b) $\log_b(x-y) \neq \log_b(x) - \log_b(y)$

    (c) $\log_b\left(\frac{x}{y}\right) \neq \frac{\log_b(x)}{\log_b(y)}$

43. The Henderson-Hasselbalch Equation: Suppose HA represents a weak acid. Then we have a reversible chemical reaction

$$HA \rightleftharpoons H^+ + A^-.$$

The acid disassociation constant, K_a, is given by

$$K_\alpha = \frac{[H^+][A^-]}{[HA]} = [H^+]\frac{[A^-]}{[HA]},$$

where the square brackets denote the concentrations just as they did in Exercise 77 in Section 6.1. The symbol pK_a is defined similarly to pH in that p$K_a = -\log(K_a)$. Using the definition of pH from Exercise 77 and the properties of logarithms, derive the Henderson-Hasselbalch Equation which states

$$\text{pH} = \text{p}K_a + \log\frac{[A^-]}{[HA]}$$

44. Research the history of logarithms including the origin of the word 'logarithm' itself. Why is the abbreviation of natural log 'ln' and not 'nl'?

45. There is a scene in the movie 'Apollo 13' in which several people at Mission Control use slide rules to verify a computation. Was that scene accurate? Look for other pop culture references to logarithms and slide rules.

Checkpoint Quiz 6.2

1. Find the exact value of the following logarithms:

   (a) $\log_3\left(\frac{1}{81}\right)$ (b) $\log(0.001)$ (c) $\ln\left(\sqrt{e^5}\right)$

2. Write as a single logarithm: $3\log_2(x) - \log_{\frac{1}{2}}(y) + 2$

For worked out solutions to this quiz, click the link below:

- Quiz Solution

6.2.2 ANSWERS

1. $3\ln(x) + 2\ln(y)$
2. $7 - \log_2(x^2 + 4)$
3. $3\log_5(z) - 6$
4. $\log(1.23) + 37$
5. $\frac{1}{2}\ln(z) - \ln(x) - \ln(y)$
6. $\log_5(x - 5) + \log_5(x + 5)$
7. $3\log_{\sqrt{2}}(x) + 4$
8. $-2 + \log_{\frac{1}{3}}(x) + \log_{\frac{1}{3}}(y - 2) + \log_{\frac{1}{3}}(y^2 + 2y + 4)$
9. $3 + 3\log(x) + 5\log(y)$
10. $2\log_3(x) - 4 - 4\log_3(y)$
11. $\frac{1}{4}\ln(x) + \frac{1}{4}\ln(y) - \frac{1}{4} - \frac{1}{4}\ln(z)$
12. $12 - 12\log_6(x) - 4\log_6(y)$
13. $\frac{5}{3} + \log(x) + \frac{1}{2}\log(y)$
14. $-2 + \frac{2}{3}\log_{\frac{1}{2}}(x) - \log_{\frac{1}{2}}(y) - \frac{1}{2}\log_{\frac{1}{2}}(z)$
15. $\frac{1}{3}\ln(x) - \ln(10) - \frac{1}{2}\ln(y) - \frac{1}{2}\ln(z)$
16. $\ln(x^4y^2)$
17. $\log_2\left(\frac{xy}{z}\right)$
18. $\log_3\left(\frac{x}{y^2}\right)$
19. $\log_3\left(\frac{\sqrt{x}}{y^2z}\right)$
20. $\ln\left(\frac{x^2}{y^3z^4}\right)$
21. $\log\left(\frac{x\sqrt{y}}{\sqrt[3]{z}}\right)$
22. $\ln\left(\sqrt[3]{\frac{z}{xy}}\right)$
23. $\log_5\left(\frac{x}{125}\right)$
24. $\log\left(\frac{1000}{x}\right)$
25. $\log_7\left(\frac{x(x-3)}{49}\right)$
26. $\ln\left(x\sqrt{e}\right)$
27. $\log_2\left(x^{3/2}\right)$
28. $\log_2\left(x\sqrt{x-1}\right)$
29. $\log_2\left(\frac{x}{x-1}\right)$
30. $7^{x-1} = e^{(x-1)\ln(7)}$
31. $\log_3(x + 2) = \frac{\log(x+2)}{\log(3)}$
32. $\left(\frac{2}{3}\right)^x = e^{x\ln\left(\frac{2}{3}\right)}$
33. $\log(x^2 + 1) = \frac{\ln(x^2+1)}{\ln(10)}$
34. $\log_3(12) \approx 2.26186$
35. $\log_5(80) \approx 2.72271$
36. $\log_6(72) \approx 2.38685$
37. $\log_4\left(\frac{1}{10}\right) \approx -1.66096$
38. $\log_{\frac{3}{5}}(1000) \approx -13.52273$
39. $\log_{\frac{2}{3}}(50) \approx -9.64824$